DEPLETION OF AN OIL DEPOSIT IN AN ELASTICALLY COMPRESSIBLE FISSURED-POROUS FORMATION

V. S. Nustrov UDC 532.546

It is proposed that the drop in the fluid pressure in a reservoir results in a significant deformation of the main channels (cracks).

1. Hydrodynamic tests on wells and theoretical studies [1-7] show that deformations occurring in the process of development bring about significant changes in the percolation-capacity properties of fissured-porous reservoirs. First the cracks are pressed together, right up to the point of closure (see, for example, [8, 9]); in the general case the region of percolation is separated into zones with open and closed cracks [10] (far away from and close to the well, respectively). The problem of the depletion of fissured-porous fised, taking into account the motion of the front of closure of cracks, was first studied in [11]. Only the equation of percolation for Leibenzon's function with different formation parameters before and after closure of the cracks was employed. In what follows the analysis of the depletion process is based on the representation of a fissured-porous medium by a collection of two interpenetrating continua [12], modified in [13] in order to take into account more fully the strong dependence of the percolation-capacity properties of the medium on its stressed state. The process is studied in the natural regime up to the moment of closure of the cracks near the well; this formulation of the problem of the motion of the front of closure of the cracks is not studied here. Other percolation problems are studied in [4-16] on the basis of the model of [13].

One-dimensional percolation of a liquid in an elastic fissured-porous medium is described by the system of equations [13].

$$a\frac{\partial\varphi_1}{\partial\omega}=\nabla(\varphi_1^3\nabla\varphi_1)+\varphi_2-\varphi_1,\quad \frac{\partial\varphi_2}{\partial\omega}=\varepsilon\nabla^2\varphi_2-\varphi_2+\varphi_1, \tag{1}$$

$$\varphi_i=(p_i-\sigma)/(p^\circ-\sigma)\quad (i=1,\ 2),\quad \xi=r(\varkappa_1\tau)^{-\frac{1}{2}},\quad \omega=t/\tau;$$
$$a=\overset{\circ}{m}_1[\delta(p^\circ-\sigma)]^{-1},\quad \varepsilon=\varkappa_2/\varkappa_1=\overset{\circ}{k}_2/\overset{\circ}{k}_1, \tag{2}$$
$$\varkappa_1=\overset{\circ}{k}_1/(\mu^\circ\delta),\quad \varkappa_2=\overset{\circ}{k}_2/(\mu^\circ\delta),\quad \delta=\overset{\circ}{m}_2(K_\rho^{-1}+K_m^{-1}).$$

In Eqs. (1) and (2) $\varepsilon \ll 1$ is the ratio of the permeability of blocks and cracks with an initial formation pressure p^0; the mass transfer is assumed to be quasistationary; and, the parameter a can be of any order (for the model of [12] $a \ll 1$). The nonlinear term in Eqs. (1) characterizes the eleastic deformation of the cracks.

The system (1) meaningful only for $p_1 > \delta$ (the pressure in the cracks is greater than some critical value). For $p_1 \le \sigma$ the cracks are closed and percolation of the liquid in this zone occurs along blocks (equation of the elastic regime), and the conditions of joining hold on the unknown boundary between the zones (where $p_1 = \sigma$) [14]. According to the experimental data of [8, 10], the quantity σ can be 10-25% lower than the starting formation pressure p^0. In [11] it is pointed out that the value of σ is close to the hydrostatic pressure.

For the depletion problem the solution of the system (1) must satisfy on the contour the condition

---

A. M. Gor'kii Ural State University, Sverdlovsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 58, No. 1, pp. 82-88, January, 1990. Original article submitted June 23, 1988.

0022-0841/90/5801-0069$12.50

$$\varphi_1^3 \partial\varphi_1/\partial\xi = \partial\varphi_2/\partial\xi = 0, \quad \xi = \xi_R = R(\varkappa_1\tau)^{-1/2}. \tag{3}$$

2. We assume that the constant bottom hole pressure is given in the well:

$$\varphi_1 = \varphi_2 = \varphi_0 > 0, \quad \xi = \xi_0 = r_0(\varkappa_1\tau)^{-1/2}. \tag{4}$$

The pressure profiles

$$\varphi_i = -2\gamma_i\xi_R^2 \ln(\xi/\xi_0) + \varphi_0 + \gamma_i(\xi^2 - \xi_0^2) \quad (i = 1,\ 2), \tag{5}$$

satisfy the conditions (3) and (4). Here the functions $\gamma_i = \gamma_i(\omega)$ are determined from the integral relations, obtained by integrating (1) over ξ for ξ_0 to ξ_R taking into account Eqs. (3)-(5); these relations express the balance of mass (the procedure is given in [15, 16]). Omitting the calculations, we present the final equations:

$$\begin{gathered} a\,d\gamma_1/d\omega = b_1\gamma_1 + \gamma_2 - \gamma_1, \quad d\gamma_2/d\omega = b_2\gamma_2 - \gamma_2 + \gamma_1, \\ b_1 = 2\varphi_0^3(\xi_R^2 - \xi_0^2)/c \approx 2\varphi_0^3\xi_R^2/c, \quad b_2 = 2\varepsilon(\xi_R^2 - \xi_0^2)/c \approx 2\varepsilon\xi_R^2/c, \\ c = \frac{3}{4}\xi_R^4 + \frac{1}{4}\xi_0^4 + \xi_R^4 \ln(\xi_0/\xi_R) - \xi_0^2\xi_R^2 \approx \xi_R^4\left[\frac{3}{4} + \ln(\xi_0/\xi_R)\right] < 0, \end{gathered} \tag{6}$$

whence for the initial conditions $\varphi_i(\xi_R,\ 0) = 1$ we find

$$\begin{gathered} \gamma_1(\omega) = (\lambda_1 - b_2 + 1)\,u_1(\omega) + (\lambda_2 - b_2 + 1)\,u_2(\omega), \\ \gamma_2(\omega) = u_1(\omega) + u_2(\omega), \\ u_1(\omega) = \frac{\gamma(0)}{\lambda_1 - \lambda_2}(b_2 - \lambda_2)\exp(\lambda_1\omega), \\ u_2(\omega) = -\frac{\gamma(0)}{\lambda_1 - \lambda_2}(b_2 - \lambda_1)\exp(\lambda_2\omega), \\ \gamma(0) = (1 - \varphi_0)/n, \quad n = \xi_R^2 - \xi_0^2 - 2\ln(\xi_R/\xi_0) \approx \xi_R^2 - 2\ln(\xi_R/\xi_0), \end{gathered} \tag{7}$$

where the roots of the characteristics equation are given by

$$\begin{gathered} \lambda_{1,2} = (2a)^{-1}\{g \pm [g^2 - 4a(b_1b_2 - b_1 - b_2)]^{1/2}\}, \\ g = b_1 - 1 + a(b_2 - 1), \end{gathered} \tag{8}$$

and are real ang negative ($\lambda_1 \neq \lambda_2$).

Using Eqs. (6)-(8) we can show that $\gamma_i(\omega) < 0$ for $0 \leq \omega\ \infty$, the coefficients in Eq. (7) $\lambda_1 - b_2 + 1$, $\lambda_2 - b_2 + 1$, differ from 0, and one of the coefficients $b_2 - \lambda_2$, $b_2 - \lambda_1$ can vanish if

$$\varepsilon = 4\varphi_0^3/a. \tag{9}$$

In this case the functions $\gamma_i(\omega)$ are identical, as a result of which the pressure distributions (5) in the cracks and the blocks are also identical, and therefore the fissured-porous medium is equivalent to a uniform porous medium. The condition (9) will hold either with low bottom hole pressure φ_0 (the cracks are equivalent to pores in the blocks owing to their strong compression) or for sufficiently large values of the parameter a (in this case the percolation is slow [15] and the cracks do not play their usal role of basic channels). Therefore the arguments presented are valid if $\varepsilon < 4\varphi_0^3/a$, (in what follows we assume that this inequality holds), and in this case $\gamma_1(\omega) \neq \gamma_2(\omega)$ for $0 < \omega < \infty$. In the limit $\omega \to \infty$ $\gamma_i(\omega) \to 0$ φ_1, $\varphi_2 \to \varphi_0$.

Using Eqs. (5)-(7) we shall write an expression for mass transfer between the blocks and cracks:

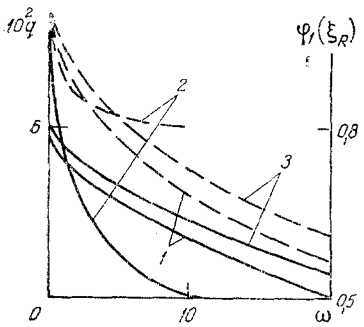

Fig. 1. The drop in the production (solid lines) and the contour pressure (broken lines) for a fissured-porous reservoir: $\varphi_0 = 0.5$ (1); 0.8 (2); the numerical solution for $\varphi_0 = 0.5$ (3).

$$\varphi_2 - \varphi_1 = [\xi^2 - \xi_0^2 - \xi_R^2 \ln (\xi/\xi_0)^2][\gamma_2(\omega) - \gamma_1(\omega)].$$

At each moment in time the flow of liquid from blocks into cracks increases monotonically as the distance from the well increases ($d(\varphi_2-\varphi_1)/d\xi \geqslant 0$, the equality to zero holds only if $\xi=1$), $\varphi_2-\varphi_1 \to 0$ as $\omega \to \infty$.

According to Eqs. (5) and (7) the production of the well and the contour pressure in the cracks have the form (Fig. 1):

$$q/(2\xi_R^2) = -[\varphi_0^3\gamma_1(\omega) + \varepsilon\gamma_2(\omega)] \approx -\varphi_0^3\gamma_1(\omega),$$
$$\varphi_1(\xi_R, \omega) = \varphi_0 + n\gamma_1(\omega), \qquad (10)$$

where the term $\varepsilon\gamma_2(\omega)$ in the first formula must be taken into account at the end of the depletion process. The curves in Fig. 1 were constructed for the following values of the parameters:

$$r_0 = 0{,}1 \text{ m}, \quad R = 200 \text{ m}, \quad h = 10 \text{ m}, \quad \mathring{m}_1 = 0{,}15\ \%, \quad \mathring{m}_2 = 15\ \%,$$
$$p^\circ - \sigma = 10^7 \text{ N/m}^2, \quad \mathring{k}_1 = 10^{-12} \text{ m}^2, \quad \mathring{k}_2 = 1{,}7\cdot 10^{-14} \text{ m}^2,$$
$$\mu^\circ = 10^{-3} \text{ Nsec/m}^2, \quad \tau = 3600 \text{ sec} \quad K_\rho^{-1} + K_m^{-1} = 10^{-9} \text{ m}^2/\text{N}$$

which corresponds in Eq. (1) to $a = 1$ and $\varepsilon = 0.017$. In all figures, if the dimension is not explicitly indicated, dimensionless parameters are employed, and the results of a numerical calculation, obtained using an implicit scheme [17], are presented to monitor the accuracy.

From Eq. (10) with $\omega = 0$ we find that the starting production is a nonmonotonic function of φ_0: q(0) has a maximum value at $\varphi_0 = 0.75$, and $q(0) = 0$ at $\varphi_0 = 0$, $\varphi_0 = 1$.

To evaluate the effect of the compressibility of the cracks on the deplation process Fig. 2 shows as or function of the dimensional time the relative production of the well q(t)/q(0) and the contour pressure in the case of a fissured reservoir (nonporous matrix), when percolation is described by the equation

$$\partial\varphi/\partial t = \varkappa\nabla(\varphi^3\nabla\varphi), \quad \varkappa = (p^\circ - \sigma)\, k^\circ/(m^\circ\mu R^2).$$

The ratio of the starting permeability and porosity of the cracks was varied in the range $k^\circ/m^\circ \sim (10^{-9}\text{-}10^{-11})$ m², which corresponded to the intervals [4] $k^\circ \sim (10^{-13}\text{-}10^{-12})$ m², $m^\circ \sim (0.01\text{-}10)$ %. For example $k^\circ/m^\circ \sim (10^{-11})$ m², with $k^0 \sim 10^{-12}$ m² (1 D), $m^\circ \sim 10\%$. As the ratio k°/m° increases by an order of magnitude the time over which the contour pressure drops to the bottom hole pressure φ_0 decreases approximately by an order magnitude, and this process is strongly affectd by the value of φ_0 . For example, for $\varphi_0 = 0.25$ (the cracks are strongly compressed) the indicated time is equal to 300 days with $k^\circ/m^\circ \sim 10^{-10}$ m² and 30 days with $k^\circ/m^\circ \sim 10^{-9}$ m². For $\varphi_0 = 0.75$ we obtain 10 days and 1 day, respectively. The drop in the relative production also slows down as the compression of the cracks increases (in Fig. 2a the values of q (0) for different values of φ_0 are different). For comparison Fig. 2a shows the drop in the relative production in the case of undeformable cracks (the initial opening remains unchanged).

3. For a constant well production (we neglect the flow along blocks) $q = \xi\varphi_1^3\partial\varphi_1/\partial\xi$, $\xi = \xi_0$, we transform in the sytem (1) to zero initial conditions, introducing the function $\psi_i = \varphi_i - 1$. We take the pressure profiles in the form [18, 16] (the first phase of the process):

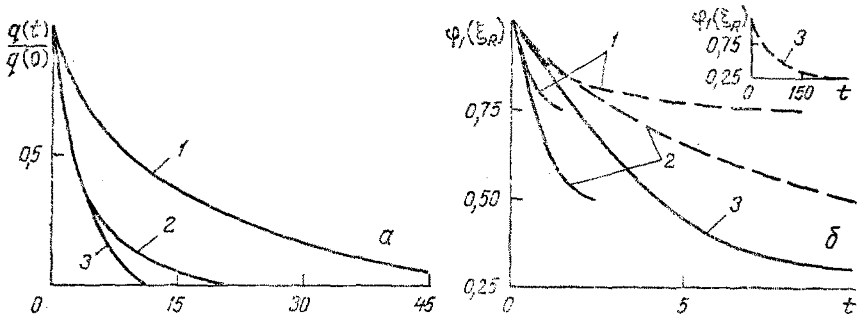

Fig. 2. The drop in the relative production (a) and the contour pressure (b) for a fissured reservoir (the dimension of the time is days): a) $p_0/p^\circ = 0.2$ (1), 0.8 (2), undeformable medium (3); b) $\varphi_0 = 0.75$ (1), 0.5 (2), 0.25 (3), $k^\circ/m^\circ \sim 10^{-9}$ m^2 (solid lines) and 10^{-10} m^2 (broken lines).

$$(\psi_i + 1)^4 = 1 + q[1 + \ln(\xi/l_i) - \xi/l_i], \tag{11}$$

where $\ell_i = \ell_i(\omega)$ are the boundaries of the zones of disturbance along cracks and blocks, determined from the integral relations corresponding to (1). We introduce the epxression $\ell_1 = \ell_1(\omega)$ [16]

$$(2I - 1)\, l_1^2(\omega) = f(\omega), \quad f(\omega) = 2q[\exp(s\omega) - 1 - (1 + a)\omega](1 + a)^{-2},$$
$$I = \int_0^1 u[1 + q(1 + \ln u - u)]^{1/4} du, \; s = -1 - \frac{1}{a}, \tag{12}$$

where the integral I is calculated numerically, and $f(\omega) \leq 0$ for $\omega \geq 0$ (the equality to zero occurs only if $\omega = 0$). The moment of termination of the first phase $\leftarrow = \omega_1$ is determined from the equation $\ell_1(\omega_1) = 1$.

At the second phase of the process (the condition (3) $\partial\varphi_1/\partial\xi = 0, \xi = \xi_R$ is employed) the expression (11) for ψ_2 remains in force and the function ψ_1 is sought in the form

$$(\psi_1 + 1)^4 = q[\ln(\xi/\xi_R) - \xi/\xi_R] + \beta(\omega). \tag{13}$$

Omitting the calculations (see [16]) we present an equation for $\beta(\omega)$, obtained from integral relations of the second phase:

$$2\int_{\xi_0}^{\xi_R} \xi\psi_1 d\xi = f(\omega). \tag{14}$$

The function $\beta(\omega)$ appears in the integrand in Eq. (14), and this makes it difficult to determine it. A numerical calculation at this stage is pointless, since the approximate solution Eqs. (11) and (13) is constructed in order to obtain simple estimates of the depletion of the field.

Let us assume that for cracks the volume—averaged pressure agrees quite well with the contour pressure (it is checked analogously to the case of a gaseous porous collector [19]). Then in Eq. (12) $I \approx 0.5$ and the first phase of the process is neglected, $\omega_1 \approx 0$ (correspondingly the depletion starts with the second phase). On the basis of this proposition, taking into account the dependence of the porosity of the cracks [13] $m = m^\circ(\psi + 1)$, we write the equality

$$\int_{\xi_0}^{\xi_R} \xi(\psi_1 + 1)^2 d\xi = [\psi_1(\xi_R, \omega) + 1] \int_{\xi_0}^{\xi_R} \xi(\psi_1 + 1)\, d\xi. \tag{15}$$

The integrals in Eq. (15) can be regarded as written for a gas (with the pressure $\psi_1 + 1$) in the case of a porous reservoir. Practical calculations for gas fields have shown [13] that the value of the squared pressure averaged over the volume of the pores is approximately equal to the squared average pressure, and therefore the equality (15) assumes the form

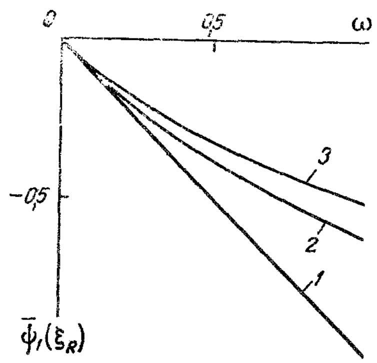

Fig. 3. The drop in the contour pressure for a fixed production rate of the well, $a = 1$: 1) fissured reservoir, 2) fissured-porous reservoir, 3) numerical solution for fissured-porous reservoir.

$$2\int_{\xi_0}^{\xi_R} \xi(\psi_1 + 1)\,d\xi = \psi_1(\xi_R, \omega) + 1,$$

whence, using Eq. (14), we find the function $\beta(\omega)$.

$$\beta(\omega) = q + [f(\omega) + \xi_R^2 - \xi_0^2]^4 \approx q + [f(\omega) + \xi_R^2]^4. \tag{16}$$

For the contour pressure we obtain from Eqs. (13) and (16) the expression

$$\bar{\psi}_1(\xi_R) = f(\omega)/(2q), \quad \bar{\psi}_1(\xi_R) = [\psi_1(\xi_R, \omega) + 1 - \xi_R^2]/(2q). \tag{17}$$

For a fissured reservoir with the equation of percolation

$$a\partial\varphi/\partial\omega = \nabla(\varphi^3\nabla\varphi)$$

the analog of the expression (17) has the form (it is identical to the result obtained for porous gas reservoirs [18, 19]):

$$\bar{\psi}(\xi_R) = -\omega/a. \tag{18}$$

For all values of the parameter a the straight line described by Eq. (18) lies below the curve described by Eq. (17), and therefore a fissured-porous reservoir is depleted more slowly than a fissured reservoir owing to the inflow of liquid from blocks (Fig. 3).

For the reservoir model studied above it is of interest to estimate the time of closure of the cracks for a well (it corresponds to the pressure $\psi_1 = -1$) with a fixed production rate. Using Eq. (16) we obtain from Eq. (13) an equation for $\omega = \omega_*$:

$$f(\omega_*) = F(q), \; F(q) = \xi_0^2 - \xi_R^2 + q^{1/4}[\xi_0/\xi_R - 1 - \ln(\xi_0/\xi_R)^{1/4}, \tag{19}$$

where $f(\omega_*) \leq 0$ and therefore $F(q)$ satisfies the condition

$$q \leqslant -\xi_R^8[1 + \ln(\xi_0/\xi_R)]^{-1}, \tag{20}$$

which for the starting data presented above gives $q \leq 0.64$.

Large values of the function $F \leq 0$ in Eq. (19) correspond to small values of ω_*. In addition, F increases as q increases. Therefore the closure time of the cracks $\omega = \omega_*$ in the second phase decreases as the production rate of the well increases.

When the condition (2) is not satisfied the cracks close during the first phase. We obtain an equation for $\omega = \omega_*$ from Eqs. (11) and (12) in the form of Eq. (19) with the function $F(q)$

$$F(q) = (2I - 1)\xi_0^2 \exp[2(1 + q)/q],$$

and the time $\omega = \omega_*$ decreases as q increases.

The equation (19) is approximate, since flow along cracks, when the cracks close, becomes comparable to a flow along blocks, which was ignored above.

Based on the foregoing analysis and comparisons with numerical calculations we can conclude that the approximations chosen for the pressure profiles describe quite well the process of depletion of an elastic fissured-porous reservoir. There are no difficulties in using the formulas presented in practical applications. The basic results are also valid for an elastic fissured reservoir.

NOTATION

p, t, r and φ, ω, ξ dimensional and dimensionless pressure, time, and coordinate; k, permeability; $\varkappa$, piezoelectric conductivity; p, porosity; μ, viscosity; σ, characteristic value of the pressure; K_ρ and K_m elastic moduli of the liquid and the blocks; r_0 and R, radius of the well and the deposit; τ, characteristic time of the fissured-porous reservoir; q, production rate; h, capacity of the reservoir; φ_0, bottom hole pressure. The indices 1 and 2 refer to cracks and blocks; ° denotes an intital value with the starting formation pressure p°.

LITERATURE CITED

1. E. S. Romm, Percolation Properties of Fissured Rocks [in Russian], Moscow (1966).
2. V. M. Dobrynin, Deformation and Changes in the Physical Properties of Oil and Gas Reservoirs [in Russian], Moscow (1970).
3. V. N. Nikolaevskii, K. S. Basniev, A. T. Gorbunov, and G. A. Zotov, Mechanics of Saturated Porous Media [in Russian], Moscow (1970).
4. V. N. Maidebor, Development of Oil Fields with Fissured Reservoirs [in Russian], Moscow (1971).
5. A. T. Gorbunov, Development of Anomalous Oil Filds [in Russian], Moscow (1981).
6. M. T. Abasov, A. T. Gorbunov, and A. Kh. Shakhverdiev, Neftepromyslovie Delo, No. 5, 3-6 (1981).
7. A. Kh. Shakhverdiev, Izv. Akad. Nauk AzSSR, Ser. Nauk Zemle, No. 1, 80-85 (1983).
8. Yu. V. Vadetskii, K. M. Obmoryshev, and B. I. Okun', Testing of Fissured Reservoirs During Drilling [in Russian], Moscow (1976).
9. I. P. Popov, Neftegaz. Geol. Geofiz., No. 9, 18-20 (1982).
10. Sh. K. Gimatudinov [Ed.], Handbook on the Design, Development, and Exploration of Oil Fields. Oil Production [in Russian], Moscow (1983).
11. M. G. Alishaev and N. Sh. Khairedinov, Izv. Akad. Nauk SSR, MZhG, No. 6, 78-83 (1985).
12. G. I. Barenblatt and Yu. P. Zheltov, Dokl. Akad. Nauk SSSR, 132, No. 3, 545-548 (1960).
13. Yu. A. Buevich, Inzh.-Fiz. Zh., 46, No. 4, 593-600 (1984).
14. Yu. A. Buevich and V. S. Nustrov, Inzh.-Fiz. Zh., 48, No. 6, 943-950 (1985).
15. V. S. Nustrov, Inzh.-Fiz. Zh., 53, No. 4, 617-624 (1987).
16. V. S. Nustrov, Inzh.-Fiz. Zh. 53, No. 994-1000 (1987).
17. A. A. Samarskii and I. M. Sobol', Zh. Vychisl. Mat. Mat. Fiz., 3, No. 4, 702-712 (1963).
18. G. I. Barenblatt, V. M. Entov, and V. M. Ryzhik, Theory of Nonstationary Percolation of Liquid and Gas [in Russian], Moscow (1972).
19. B. B. Lapuk, Dokl. Akad. Nauk SSSR, 58, No. 1, 33-36 (1947).